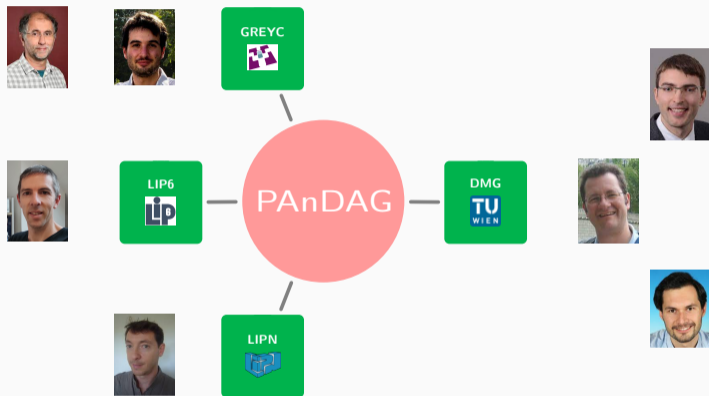


Parameter Analysis in Directed Acyclic Graphs



ANR – FWF project



Antoine Genitrini

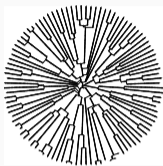
Thursday, 30 November 2023.

Discrete structures from fundamental Computer Science

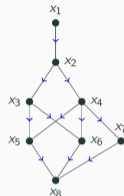
Proof-tree

$$\begin{array}{c}
 \frac{\frac{-F, F, -G \vdash F}{\quad} Ax \quad \frac{\frac{\quad}{-F, F, -G \vdash \perp} \neg e}{-F, F, -G \vdash \perp} \neg e}{\quad} \\
 \frac{\frac{\frac{-F, F \vdash G}{\quad} Abs}{-F \vdash F \rightarrow G} \rightarrow i}{\vdash -F \rightarrow (F \rightarrow G)} \rightarrow i
 \end{array}$$

Tree



Poset



λ -term



- **Counting / Enumerating** according to given parameters
- **Asymptotic analysis** of **random** objects

Systematic decomposition, *grammars*
functional equations solving, ...

\longleftrightarrow **Enumerative combinatorics**

Correspondences, *entropic random generation*, ...

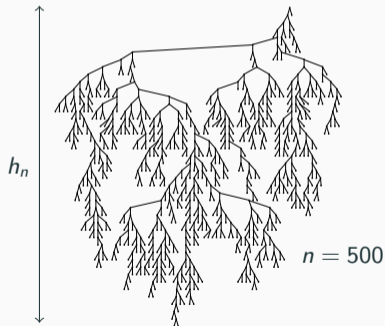
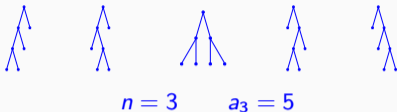
\longleftrightarrow **Bijjective combinatorics**

Asymptotical behaviour, properties in average
typical properties, *algorithm analysis in average*, ...

\longleftrightarrow **Analytic combinatorics**

Round trip: Combinatorics \leftrightarrow Computer Science

Binary trees with n internal nodes



h_n : stack height when traversing
the tree with a depth-first search.

$$a_n = \frac{1}{n+1} \binom{2n}{n}$$

$$a_n \sim \frac{1}{\sqrt{\pi n^3}} 4^n.$$

Complexity analysis in average,
the foundations of **Analytic Combinatorics**
[Flajolet-Odlyzko'82]

$$\text{average height: } \bar{h}_n \sim 2\sqrt{n}.$$

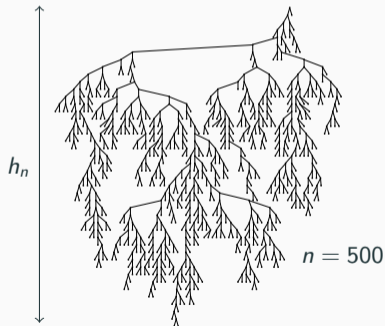
Round trip: Combinatorics \leftrightarrow Computer Science

Binary trees with n internal nodes



$n = 3$

$a_3 = 5$



$n = 500$

h_n : stack height when traversing
the tree with a depth-first search.

$$a_n = \frac{1}{n+1} \binom{2n}{n}$$

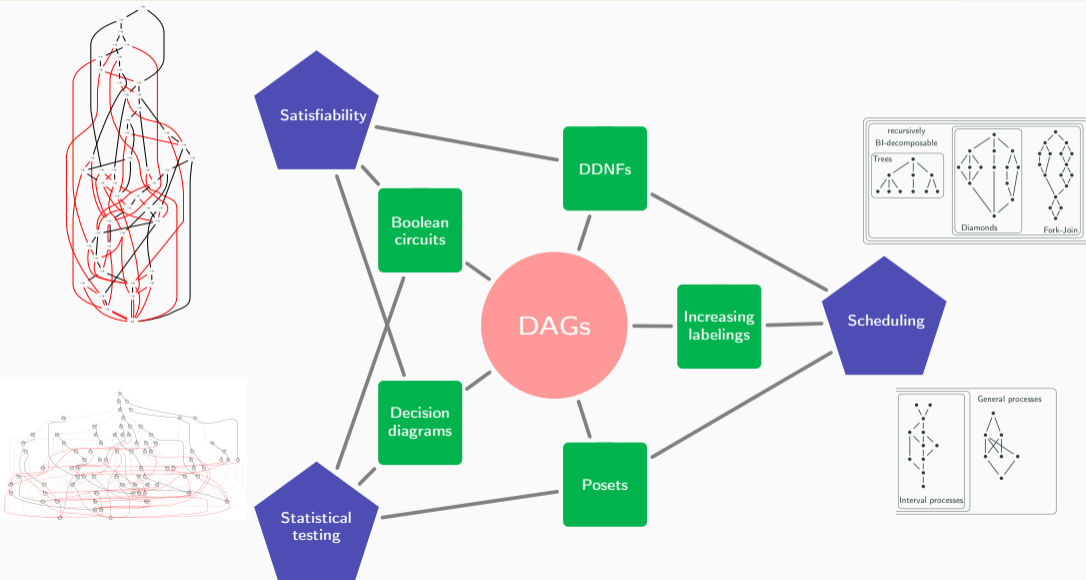
$$a_n \sim \frac{1}{\sqrt{\pi n^3}} 4^n.$$

Complexity analysis in average,
the foundations of **Analytic Combinatorics**
[Flajolet-Odlyzko'82]

$$\text{average height: } \bar{h}_n \sim 2\sqrt{n}.$$

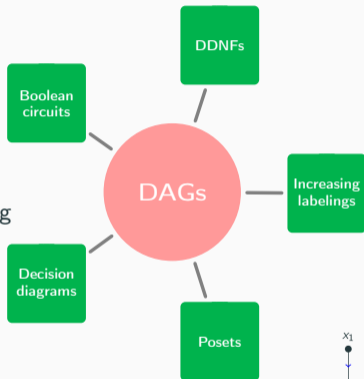
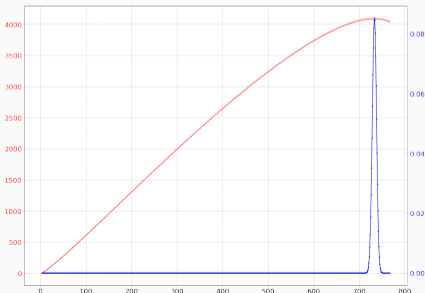
What about complex families of structures?
More complicated but that appear in applications!

Parameter Analysis in DAGs – PAnDAG project



Challenging questions

- Typical structure of a circuit
- Shannon effect
- Boltzmann sampling
- Controlled but non uniform sampling



- Combinatorial specifications of subclasses of BDDs
- Adapting Borel transform
- Analysis of growth processes
- Combinatorial explosion: design of partial exploration

